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Superluminal neutrino, flavor, and relativity

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Abstract

Modified neutrino dispersion relations, which still obey the relativity principle, can have both a superluminal (muon-type) neutrino and a luminal (electron-type) neutrino, as long as neutrino-mass effects can be neglected. The idea is to allow for flavor-dependent deformed Lorentz transformations and an appropriate hierarchy of energy scales. If OPERA's result on a superluminal velocity of the muon-neutrino is confirmed, the model has a matching superluminal velocity of the corresponding charged lepton, the muon, at equal particle energy. Assuming that this model is not already ruled out, new TeV-scale effects in the muon sector are predicted. Also discussed is a different model with a superluminal sterile neutrino propagating in the usual 4 spacetime dimensions.

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I. INTRODUCTION

Let us be radical and take OPERA's result [1] at face value (combining the quoted statistical and systematic errors in quadrature):

$$\Big(\big[v_{\nu_\mu} \big]^{(\mathrm{exp})}_{\langle c\,|\mathbf{p}|\rangle = 17\ \mathrm{GeV}} - c \Big) \Big/ c = (2.4 \pm 0.4) \times 10^{-5}\,, \tag{1}$$

with c the velocity of light *in vacuo* and v the inferred time-of-flight velocity of the neutrino. A similar value, but with larger errors, has been reported earlier by MINOS [2]. We are, of course, well aware of the reservations expressed regarding the claimed OPERA result (some papers are listed in Ref. [3]), but the purpose, here, is to set a problem which is as difficult as possible, taking the surprisingly large magnitude indicated by (1) as given.

Using the observed neutrino burst from supernova SN1987a [4], interpreted as a burst of electron-type antineutrinos, we also have the following bound [5]:

$$\Big| \big[v_{\overline{\nu}_e} \big]^{(\mathrm{exp})}_{c\,|\mathbf{p}| = 10\ \mathrm{MeV}} - c \Big| \Big/ c \lesssim 2 \times 10^{-9}\,. \tag{2}$$

The challenge is to find an explanation that can help resolve the apparent discrepancy between these two experimental numbers, assumed to be correct. There are, however, two obstacles. First, there appear to be catastrophic vacuum-Cherenkov-type energy losses ($\nu_\mu \to \nu_\mu + Z^0 \to \nu_\mu + e^- + e^+$) for the neutrinos traveling from CERN to the Gran Sasso Laboratory [6]. Second, similar unacceptable threshold effects may occur for the pion-decay process ($\pi^+ \to \mu^+ + \nu_\mu$) responsible for the production of the muon-neutrinos at CERN [7, 8].

Both obstacles can be circumvented [9] if we assume that *deformed* Lorentz transformations apply to the modified neutrino dispersion relations, that is, if *relativity still holds* (further references on deformed Lorentz transformations and the role of relativity can be found in, e.g., Refs. [9, 10]). In that case, the nonstandard vacuum-Cherenkov-type decay processes are simply absent, which has been established [9] by a general argument and a specific calculation in a toy model. The modification of the pion-decay process has been argued to be absent as well.

The toy model of Ref. [9], however, does not indicate how different neutrino species are to be treated, let alone the other particles of the standard model. Here, we implement a suitable modification inspired by another type of model [11–13], in order to find a "relativistic" model which satisfies both (1) and (2). Henceforth, we set $c = 1$.

II. NEUTRINO ANSATZ

The crux of our suggestion is to arrange for deformed Lorentz transformations which are not mass dependent as in Ref. [9] but flavor dependent. (This is perhaps somewhat counter-intuitive, but, in the end, we do not know how the Lorentz deformation arises dynamically.) Then, the only mixing angles and phase which enter the game are the standard ones related to the mass sector (θ_{32}, θ_{21}, θ_{13}, and δ).

Specifically, take the following three neutrino dispersion relations for the weak-interaction states (flavor label $f = e,\ \mu,\ \tau$):

$$E^2 - p^2 - 2\,E^2\,p^2/(\overline{M}_f)^2 \;=\; (\widetilde{m}_f)^2\,, \tag{3}$$

with 3-momentum norm $p \equiv |\mathbf{p}|$ and effective mass values $\widetilde{m}_f$ (in terms of mass eigenvalues m_n and mixing angles θ_{32}, θ_{21}, and θ_{13}). As mentioned in the Introduction, the important point is that relativity still holds, even though the relevant Lorentz transformations are deformed and flavor dependent.

The particular flavor-dependent generators $\mathcal{N}_j^{(f)}$ of Lorentz boosts can be simply extracted from Eqs. (8) and (9) in Ref. [9]:

$$\delta_j^{(f)} E \;\equiv\; \left[\mathcal{N}_j^{(f)},\, E\,\right] = p_j \left(1 + p^2/(\overline{M}_f)^2 + 2\,E^2/(\overline{M}_f)^2\right), \tag{4a}$$

$$\delta_j^{(f)} p_k \;\equiv\; \left[\mathcal{N}_j^{(f)},\, p_k\,\right] = \delta_{j,k} \left(1 - p^2/(\overline{M}_f)^2\right) E\,. \tag{4b}$$

The left-hand side of (3) is then invariant to order $1/(\overline{M}_f)^2$. The right-hand side is a scalar and trivially invariant under these transformations. By adding higher-order terms to (4) the invariance of (3) can be extended to order $1/(\overline{M}_f)^4$ or more. For given modified dispersion relation, the exact Lorentz-boost generators can be obtained by using the general method of Ref. [10]. Needless to say, (3) is manifestly invariant under rotations of the 3-momentum $\mathbf{p}$. An important consequence of the nonlinear realization (4) is that the addition formula of particle energies is modified [9, 10].

From now on, we neglect neutrino-mass effects (they will be considered at the end of the paper, in the Appendix). The model (3), with all masses $\widetilde{m}_f$ set to zero, satisfies (1) and (2) by taking

$$(15\,\overline{M}_1)^{-2} \lesssim (\overline{M}_2)^{-2} \sim (6\ \text{TeV})^{-2}\,, \tag{5}$$

with undetermined $(\overline{M}_3)^{-2}$, for the moment. Assuming that an interacting theory can be constructed (a first step will be set in the next section), the "Alice-and-Bob" argument of Sec. II in Ref. [9] also makes clear that there can be no vacuum-Cherenkov-type energy losses in this "relativistic" model. This is in contrast to the vacuum-Cherenkov-type energy losses [6] in models with a preferred frame (e.g., the dynamical models of Refs. [12–14]).

III. GENERALIZED ANSATZ

As it stands, the modified dispersion relations (3) for $\widetilde{m}_f = 0$ apply only to the neutral leptons (ν_e, ν_μ, ν_τ), the charged leptons ($e^\pm$, $\mu^\pm$, $\tau^\pm$) having, most likely, more or less standard Lorentz-invariant dispersion relations. This implies that the previous discussion holds after the spontaneous breaking of the $SU(2)_L \times U(1)_Y$ gauge symmetry (a similar remark applies to a Fermi-point-splitting model of Lorentz violation [12]). We need to find a proper gauge-invariant generalization of (3). Such a gauge-invariant generalization may be one ingredient of the final interacting theory, which, for the moment, remains elusive. Anyway, let us start by looking for this one possible ingredient, the gauge-invariant generalization of (3).

Consider the 3×15 Weyl fermions of the standard model (SM) and three additional right-handed neutrinos [singlets under $SU(2)$ and $SU(3)$ and with zero $U(1)$ hypercharges], where all masses are neglected. These fermions have a species label $a \in \{1, \ldots, 16\}$ and

family label $f \in \{1,2,3\}$. In order to be specific (and consistent with what was used in the previous section), we identify $f=1$ with the electron family, $f=2$ with the muon family, and $f=3$ with the tau family. The combined label (a,f) will be called the flavor label.

For completeness, we give the SM representations of the 16 left- and right-handed Weyl fermions of each family:

$$
\begin{aligned}
L &: \Big[\,(3,2)_{1/3}\,\Big]_{\text{quarks}} + \Big[\,(1,2)_{-1}\,\Big]_{\text{leptons}}\,, \\
R &: \Big[\,(3,1)_{4/3} + (3,1)_{-2/3}\,\Big]_{\text{quarks}} + \Big[\,(1,1)_{-2} + (1,1)_{0}\,\Big]_{\text{leptons}}\,,
\end{aligned} \tag{6}
$$

where the entries in parentheses denote $SU(3)$ and $SU(2)$ irreducible representations and the suffix the value of the $U(1)$ hypercharge Y. The electric charge Q is given by the combination $Y/2+I_3$, with I_3 the weak isospin from the diagonal Hermitian generator T_3 of the $SU(2)$ Lie algebra.

For these 48 Weyl fermions (labeled by $a \in \{1,\ldots,16\}$ and $f \in \{1,2,3\}$), the massless dispersion relations (3) can be generalized in a relatively simple way:

$$E_{a,f}^2 - p_{a,f}^2 - 2\,E_{a,f}^2\,p_{a,f}^2/(M_f)^2 \;=\; 0\,, \quad \text{for } \; Y_{a,f} \in \{-1,\,-2,\,0\}\,, \tag{7a}$$

$$E_{a,f}^2 - p_{a,f}^2 \;=\; 0\,, \quad \text{for } \; Y_{a,f} \notin \{-1,\,-2,\,0\}\,, \tag{7b}$$

so that only the leptons have modified dispersion relations (the quarks have fractional hypercharge values). The particular generalization presented in (7) is, of course, not unique and two alternatives will be given later in this section. Furthermore, there may very well appear higher-order terms in (7a) carrying factors $1/(M_f)^{2n}$ for $n \geq 2$.

Just as in Sec. II, the dispersion relations (7) are invariant to order $1/(M_f)^2$ under Lorentz boosts with the following generators $\mathcal{N}_j^{(a,f)}$ [having raised the flavor label (a,f) for convenience and omitting this label on the energy E and momentum p in the expressions below]:

$$\delta_j^{(a,f)} E \;\equiv\; \big[\,\mathcal{N}_j^{(a,f)},\, E\,\big] = p_j\,\big(1 + \Delta^{(a,f)}\,p^2/(M_f)^2 + \Delta^{(a,f)}\,2\,E^2/(M_f)^2\big)\,, \tag{8a}$$

$$\delta_j^{(a,f)} p_k \;\equiv\; \big[\,\mathcal{N}_j^{(a,f)},\, p_k\,\big] = \delta_{j,k}\,\big(1 - \Delta^{(a,f)}\,p^2/(M_f)^2\big)\,E\,, \tag{8b}$$

using the flavor factor

$$\Delta^{(a,f)} \;\equiv\; \delta_{Y^{(a,f)},-1} + \delta_{Y^{(a,f)},-2} + \delta_{Y^{(a,f)},0} \in \{0,\,1\}\,, \tag{8c}$$

in terms of a Kronecker-type function for rational numbers $r,s \in \mathbb{Q}$: $\delta_{r,s}=1$ for $r=s$ and $\delta_{r,s}=0$ for $r \neq s$. Hence, standard Lorentz generators apply to fermions with fractional hypercharge values (quarks) and nonstandard generators to fermions with integer hypercharge values (leptons).

As the Lorentz invariance of the electron sector has been verified to a high degree of accuracy, we take

$$0 = (M_1)^{-2} \ll (M_2)^{-2} \sim (6\ \text{TeV})^{-2}\,, \tag{9}$$

where the last approximate equality results from the experimental input (1) [the supernova bound (2) is trivially satisfied for $(M_1)^2 \gg (M_2)^2$]. The $(M_3)^2$ value is, for the moment, undetermined.[1] For definiteness, we take $(M_3)^2 \gtrsim (M_2)^2$. Furthermore, it is certainly possible to have $(M_2)^2/(M_1)^2$ not zero as in (9) but sufficiently small. Still, setting $(M_2)^2/(M_1)^2 = 0$ focusses the discussion on the muon sector.

The model (7)–(9) can only correspond to an effective theory valid for energies below $\min(M_1,\, M_2,\, M_3) = \mathrm{O}(10\ \text{TeV})$. It remains to be seen what the theory looks like at energies $E \gtrsim M_2 \sim 10$ TeV. Note that if (1) were to be reduced by a factor 10^{16}, the required value of M_2 would be increased by a factor 10^8, giving a scale $M_2 \sim 10^{12}$ GeV. Considering such large scales, there is a second alternative model: all 48 Weyl fermions have the same modified dispersion relation (7a) with $M_1 = M_2 = M_3 \equiv M \gg 10$ TeV (or perhaps $M \sim 10$ TeV after all?). The interacting theory may be easier to construct for this alternative model with all fermions treated equally. An entirely different model which circumvents the interaction problem is discussed in the Appendix.

Returning to model (7)–(9), the OPERA result (1) then predicts a matching relative change of the muon velocity:

$$\Big(\big[v_{\mu^\pm} \big]^{\text{(model)}}_{c\,|\mathbf{p}|\,=\,10\ \text{GeV}} - c \Big) \Big/ c \sim 10^{-5}. \tag{10}$$

Again, indirect bounds on $|v_{\mu^\pm}/c - 1|$ relying on modified decay processes [15] may not apply to this "relativistic" model. But there are also direct laboratory experiments, for example, experiments to measure the muon anomalous magnetic moment [16, 17]. It seems unlikely that per-mill changes in the muon velocity at energies of order 10^2 GeV would have gone unnoticed, but this needs to be checked (in a theoretical framework without preferred frame and with appropriate interactions).

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Appendix A: STERILE-NEUTRINO ANSATZ

It may turn out to be impossible to embed the Lorentz violation of (7) into an interacting theory containing the SM particles and, simultaneously, avoid excessive energy losses [6] from the vacuum-Cherenkov-type process $\nu_\mu \to \nu_\mu + Z^0 \to \nu_\mu + e^- + e^+$. In that case, there exists an alternative approach, which, however, looses the prediction (10). This approach allows us to include neutrino-mass effects, which were neglected in the discussion of the main text. In fact, the neutrino eigenstates considered in the present appendix (labeled by $n \in \mathbb{N}$) are those of mass.

[1] It is also conceivable that the leptons in the tau sector ($f = 3$) have *sub*-luminal velocities. This can be achieved by replacing all occurrences of $1/(M_3)^2$ in (7) and (8) by $-1/(M_3)^2$, which corresponds to the first alternative model mentioned in the discussion below (7).

The point is that the modified dispersion relation (3) can also apply to a light sterile neutrino, which has been invoked to explain [18–20] the OPERA result (1). The three standard neutrinos ($n = 1, 2, 3$) then travel with luminal or subluminal velocities, while the light sterile neutrino (mass $m_4 \sim 0.1 - 1$ eV) is superluminal. The velocity 'v' appearing in (1) is an average of these sub- and superluminal velocities with weights depending on the mixing angles (see below). The superluminal sterile neutrino, by definition, does not couple to the Z^0 boson and does not suffer from catastrophic energy losses [6]. For further details on and challenges for such a type of phenomenological superluminal-sterile-neutrino model, see Refs. [18–21] and references therein.

The fundamental question, of course, is how the light sterile neutrino acquires a superluminal maximum velocity. One possible explanation relies on the introduction of one or more extra spatial dimensions and a braneworld with appropriate metric [19, 20, 22].

We suggest, instead, that the theory remains four-dimensional but has two sectors without direct interactions, one containing all standard-model particles (fermions, as well as gauge and Higgs bosons) with the standard linear Lorentz transformations and another containing gauge-singlet (sterile) particles with deformed nonlinear Lorentz transformations as discussed in Sec. II. Taking over the previous example (3) and temporarily reinstating the light velocity c, the dispersion relations of the four neutrino mass eigenstates ($n = 1, 2, 3, 4$) are given by

$$E^2 - c^2 p^2 - 2\, E^2 c^2 p^2 / \left(\overline{M}_n\, c^2\right)^2 \;=\; \left(m_n\, c^2\right)^2 , \tag{A1a}$$

$$1/\overline{M}_1 = 1/\overline{M}_2 = 1/\overline{M}_3 \;=\; 0 , \tag{A1b}$$

with $\overline{M}_4 = \mathrm{O}(10\ \mathrm{TeV}/c^2)$ if OPERA's result (1) sets the scale and if there is significant mixing between the $f = \mu$ flavor state and the $n = 4$ mass state. The dynamical origin of this active-sterile neutrino mixing certainly needs to be clarified. Perhaps there exists an indirect (nonperturbative) gravitational interaction between the two sectors resulting in an effective contact-interaction term in the action, which is suppressed by at least one factor $1/E_{\text{Planck}}$ (cf. Ref. [23] and references therein).

Alternatively, genuine (preferred-frame) Lorentz violation of the light sterile neutrino may come from the Fermi-point-splitting mechanism [12] or from the spontaneous breaking of Lorentz invariance (SBLI) [13], both operating in the known $3 + 1$ spacetime dimensions.

The SBLI explanation, in particular, is quite attractive as it only relies on the appearance of a fermion condensate, possibly coming from the multifermion interaction (7) of Ref. [13] applied to the gauge-singlet field of the sterile neutrino. With a timelike fermion-condensation vector $(b_\alpha) = (b_0, 0, 0, 0)$ as discussed in Ref. [13] and c temporarily reinstated, the dispersion relations of the four neutrinos are given by

$$E^2 \;=\; c^2 p^2 + \left(m_n\, c^2\right)^2 , \quad \text{for } n = 1, 2, 3, \tag{A2a}$$

$$E^2 \;\sim\; c_4^2\, p^2 + \left(m_4\, c_4^2\right)^2 , \quad \text{for } n = 4, \tag{A2b}$$

$$c_4 \;\equiv\; \left[1 - (b_0)^2\right]^{-1/2} c\, , \tag{A2c}$$

$$m_4 \;\equiv\; \left[1 - (b_0)^2\right]^{1/2} \widehat{m}_4\, , \tag{A2d}$$

where $\widehat{m}_4$ is the sterile-neutrino mass without spontaneous symmetry breaking (order parameter $b_0 = 0$) and where (A2b) holds for $E \gg |\widehat{m}_4\, c^2/b_0|$, corresponding to $E \gg 10^2$ eV for $|b_0| \sim 10^{-2}$ and $m_4 \sim \text{eV}/c^2$. The order of magnitude of $|b_0|$ is indeed 10^{-2} if OPERA's result (1) sets the scale and if there is significant $\mu 4$ mixing (a somewhat larger value of $|b_0|$ can compensate for a somewhat smaller value of the mixing angle $\theta_{\mu 4}$; see also the discussion in Refs. [19, 21]).[2]

The modified dispersion relation shown in (A2b) is only the simplest one possible. By considering higher-derivative terms in the action, SBLI can also give nonstandard terms p^{2n} with $n \geq 2$ on the right-hand side of (A2b). The resulting energy dependence of the velocity 'v' in the theoretical counterparts of Eqs. (1) and (2), together with equal mixing of the sterile neutrino and the three flavors of active neutrinos (e.g., mixing angles $\theta_{e4} = \theta_{\mu 4} = \theta_{\tau 4}$), make for phenomenologically attractive models, provided the leakage of Lorentz violation to the charged-lepton sector by quantum effects (e.g., loop corrections to the electron propagator) can be kept small enough [18].

The extra-dimensional braneworld explanation for the superluminal velocity of the sterile neutrino suggests an equal superluminal velocity of gravitational waves, which may already be in conflict with existing bounds [19]. Now, compare this expected behavior with that of the three four-dimensional mechanisms mentioned above. For the two-sector explanation, it is not really clear if the gravitational-wave velocity is modified or not. But, for the Fermi-point-splitting and SBLI explanations in their basic form, the gravitational-wave velocity is definitely equal to the speed of light, c.

As a possible explanation of the OPERA result [1], the simplest version of a superluminal-sterile-neutrino model [18, 19] is perhaps one with SBLI [13]. Further sterile-neutrino-SBLI models are discussed in Ref. [24].

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